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06MAT11

First Semester B.E. Degree Examination, December 2011
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions choosing at least two from each part.***
2. Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Select the correct answer :

i) If $y = 4^{3x}$ then y_n is
A) $4^{3x}(4\log 3)^n$ B) $(3\log 4)^n$ C) $3^{4x}(3\log 4)^n$ D) $4^{3x}(3\log 4)^n$

ii) If u and v are functions of x then $[vu]_n$ is
A) $u_n v + nc_1 u_{n-1} v_1 + \ldots\ldots + u v_n$ B) $u_n v_n + \ldots\ldots + u_{n-m} v_{n-m}$
C) $v_n u + nc_1 v_{n-1} u_1 + \ldots\ldots + v u_n$ D) $u_n + nc_1 u_{n-1} v_1 + \ldots\ldots + v_n$

iii) For $r = ae^{\theta}$, then the angle between radius vector and the tangent is
A) $\pi/2$ B) $\pi/6$ C) $\pi/4$ D) $\pi/3$

iv) The pedal equation of $r = a\sin\theta$ is
A) $p^2 a = r$ B) $pa^2 = r$ C) $pa = r$ D) $pa = r^2$. **(04 Marks)**

b. Find the n^{th} derivative of $y = e^{2x}\sin x\cos^2 x$. **(04 Marks)**

c. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$. **(06 Marks)**

d. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

2 a. Select the correct answer :

i) For $z = x\sin y + y\sin x$, $\frac{\partial^2 z}{\partial x \partial y} - (\cos y + \cos x) = \ldots\ldots$
A) sinx B) cosx C) sinxcosx D) 0

ii) If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, then the value $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is
A) 3 B) e^u C) e^{3u} D) 0

iii) If u and v are the functions of x and y and x, y are the functions of s and t then $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(s,t)}$ is
A) $\frac{\partial(u,v)}{\partial(x,y)}$ B) $\frac{\partial(x,y)}{\partial(u,v)}$ C) $\frac{\partial(u,v)}{\partial(s,t)}$ D) $\frac{\partial(s,t)}{\partial(u,v)}$

iv) For $z = f(x, y)$, if dz, dx and dy are the errors, then $\frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$ is
A) dx B) dy C) df D) dz. **(04 Marks)**

b. Define the homogeneous function f(x, y), with two examples. If u(x, y) is a homogeneous function of degree 'n' then prove that
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.$$ **(04 Marks)**

c. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. **(06 Marks)**

d. If $x + y + z = u$, $y + z = v$ and $z = uvw$, find the value of $J\left(\frac{x, y, z}{u, v, w}\right)$ **(06 Marks)**

3 a. Select the correct answer :

i) The value of $\int_0^{\pi/2} \sin^6 x \cos^5 x dx$ is

A) 6/115 B) 7/693 C) 3/512 D) 8/693

ii) The value of $\int_0^a \frac{dx}{\sqrt{a - x^2}}$ is

A) $\pi/3$ B) $\pi/4$ C) π D) 2π

iii) For $y^2(a - x)x^3$, asymptote parallel to y-axis is

A) $x = 0$ B) $x = 1/a$ C) $x = a$ D) $x = \sqrt{a}$.

iv) The curve $r = a(1 + \cos\theta)$ is symmetrical about the

A) origin B) initial line C) x-axis D) y-axis **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$. **(04 Marks)**

c. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then evaluate I_6. **(06 Marks)**

d. Trace the curve $y^2(a + x) = x^2 (a - x)$ **(06 Marks)**

4 a. Select the correct answer :

i) If P and Q be any two points on a curve $y = f(x)$ then $\lim_{Q \to P} \frac{\text{arc } PQ}{\text{chord } PQ}$ is

A) 1 B) > 1 C) < 1 D) 0

ii) For $y = c\cosh(x/c)$, the value of ds/dx is

A) cosh x/c B) sinh x/c C) cos x/c D) sin x/c.

iii) The volume of the solid generated by revolving about x-axis of the area bounded by $x = f(y)$ at $x = a$, $x \leq b$ is

A) $\int_a^b \pi y^2 dx$ B) $\int_a^b y^2 dx$ C) $\int_a^b \pi x^2 dy$ D) $\int_a^b \pi y^2 dy$

iv) The length of the arc of the curve $\theta = f(r)$ at $r = a$ and $r = b$ is

A) $\int_a^b \sqrt{r^2 + (dr/d\theta)^2}\, d\theta$ B) $\int_a^b \sqrt{1 + (dr/d\theta)^2}\, d\theta$

C) $\int_a^b \sqrt{1 + r(d\theta/dr)^2}\, d\theta$ D) $\int_a^b r(dr/d\theta)^2\, d\theta$ **(04 Marks)**

b. Find the length of one arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$. **(04 Marks)**

c. Find the volume generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line. **(06 Marks)**

d. Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1 + x^2)} dx$, using the differentiation under integral sign. **(06 Marks)**

PART – B

5 a. Select the correct answer :

i) For $dy/dx = (4x + 3y + 2)^2$, dt/dx is

A) $t^2 + 4$ B) $3t^2 + 4$ C) $4t^2 + 3$ D) $t^2 + 3$

ii) If $M(x, y)dx + N(x, y)dy = 0$, which is non exact, then $\frac{1}{M}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]$ is a function of

A) y B) xy C) x/y D) x.

iii) The integrating factor of $dy/dx + y\tan x = \cos x$ is

A) cosec x B) sin x C) cos x D) sec x

iv) In polar coordinate form $r = f(\theta)$, for the differential of orthogonal trajectory, the derivative $dr/d\theta$ is replaced by

A) $-r\frac{d\theta}{dr}$ B) $r^2\frac{d\theta}{dr}$ C) $-r^2\frac{d\theta}{dr}$ D) $-r^2\frac{dr}{d\theta}$ **(04 Marks)**

b. Solve $(x^2 + y^3 + 6x)dx + y^2x\,dy = 0$ **(04 Marks)**

c. Solve $dy/dx + y\tan x = y^3\sec x$. **(06 Marks)**

d. Find the orthogonal trajectory of $r^n \sin n\theta = a^n$, with a-parameter and solve. **(06 Marks)**

6 a. Select the correct answer :

i) If $\sum_1^{\infty} u_n$ is a series given and if $\lim_{n \to \infty} S_n$ tends to finite or infinite, then the series is

A) divergent B) convergent C) oscillatory D) p-series

ii) If $\sum u_n = \sum \frac{2^n}{n}$, then the value of $\lim_{n \to \infty} \frac{u_{n+1}}{u_n}$ is

A) 1/2 B) 2 C) $\sqrt{2}$ D) 1/3.

iii) If the value of $\lim_{n \to \infty} \sqrt[n]{u_n}$ for $\Sigma(3/2)^n n^5$ is

A) 3/2 B) 2/3 C) ½ D) 1/3.

iv) One of the conditions for an alternating series to be convergent, if $\lim_{n \to \infty} u_n$ is

A) 1 B) < 0 C) > 0 D) $= 0$ **(04 Marks)**

b. Test for convergence of $\frac{x}{3} + \frac{1\times 2}{3\times 5}x^2 + \frac{1\times 2\times 3}{3\times 5\times 7}x^3 + \ldots\ldots\ldots$ **(04 Marks)**

c. Examine the convergence of $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^2}$. **(06 Marks)**

d. State the Leibnitz theorem for absolute and conditional convergence. Discuss the convergence of the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \ldots\ldots\ldots$ for absolute and conditional convergence. **(06 Marks)**

7 a. Select the correct answer :

i) Direction cosines of z-axis are

A) (1, 1, 1) B) (1, 0, 1) C) (0, 1, 0) D) (0, 0, 1)

ii) If l_1, m_1, n_1 and l_2, m_2, n_2 are the dc's of two lines which are perpendicular then $l_1l_2 + m_1m_2 + n_1n_2$ is

A) $\pi/2$ B) -1 C) 1 D) 0

iii) The angle between the two planes $2x - 3y + z + 5 = 0$ and $x + 2y + 7z - 3 = 0$ is $\cos\theta =$

A) 9.165 B) 8.265 C) 7.875 D) 5.5

iv) The minimum perpendicular length between the two ______lines is the shortest distance.

A) parallel B) perpendicular C) skew D) intersecting **(04 Marks)**

b. Prove that the lines whose DC's are given by the relations $al + bm + cn = 0$ and $mn + ln + lm = 0$ are perpendicular, if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. **(04 Marks)**

c. Find the equation of the plane through the point (1, –1, 2) and perpendicular to the plane $x + 2y - 3z = 8$ and $2x + 3y - 2z = 5$. **(06 Marks)**

d. Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Also find the equation of the line of shortest distance. **(06 Marks)**

8 a. Select the correct answer :

i) Vector differential operator ∇ is defined as

A) $\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}$ B) $i\frac{d}{dx}+j\frac{d}{dy}+k\frac{d}{dz}$ C) $x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+z\frac{\partial}{\partial z}$ D) $i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}$

ii) The gradient of a constant is

A) constant B) 1 C) 0 D) $x + y + z$

iii) $\nabla \cdot F$ is ______when F is vector point function

A) vector point function of t B) solenoidal

C) irrotational D) scalar point function.

iv) Curl(grad ϕ) is

A) 0 B) $\nabla\phi$ C) $\nabla^2\phi$ D) 1. **(04 Marks)**

b. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $i + j + 2k$. **(04 Marks)**

c. Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)i + (x + cy + 2z)k + (bx + 2y - z)j$ is irrotational. **(06 Marks)**

d. Prove that $\nabla\cdot(\nabla\phi) = \nabla^2\phi$. Given $\phi = xy + yz + zx$. **(06 Marks)**

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